VIABILITY OF MATRIX-FRACTURE TRANSFER FUNCTIONS FOR DILUTE SURFACTANT-AUGMENTED WATERFLOODING IN FRACTURED CARBONATE RESERVOIRS

by
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A thesis submitted to the Faculty and the Board of Trustees of the Colorado School of Mines in partial fulfillment of the requirements for the degree of Doctor of Philosophy (Petroleum Engineering).

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The attached document is my PhD dissertation proposal. It contains a detailed outline of the dissertation, the objectives for the research, and a summary of completed class work.

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NOMENCLATURE

General Nomenclature

$A$ ................................................. surface area, $ft^2$

$a$ ................................................. surfactant adsorption, $\frac{\mu g}{g}$

$C_s^*$ ........................................ surfactant solution concentration, ppm

$C_s$ ........................................ surfactant concentration, ppm

$c_o$ ........................................ oil compressibility, $psi^{-1}$

$c_\phi$ ........................................ pore compressibility, $psi^{-1}$

$c_w$ ........................................ water compressibility, $psi^{-1}$

$D$ ........................................ depth from datum, $ft$

$d$ ........................................ core diameter, $inch$

EOR ........................................... Enhanced Oil Recovery

$g$ .............................................. earth gravity acceleration, $9.8 \frac{m}{s^2}$

$g_c$ ........................................ centrifuge gravity acceleration, $\frac{m}{s^2}$

$h$ ............................................ fluid head, $ft$

IFT .......................................... Interfacial tension

$I$ .............................................. Identity matrix

$J$ .............................................. Jacobian matrix

LD ........................................... Long Dolomite

LMA ......................................... Levenberg-Marquardt Algorithm

$k$ .............................................. permeability, $md$
<table>
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<tr>
<td>$k_{ro}$</td>
<td>oil relative permeability</td>
</tr>
<tr>
<td>$k_{rw}$</td>
<td>water relative permeability</td>
</tr>
<tr>
<td>$k_{ro}^*$</td>
<td>oil relative permeability end point</td>
</tr>
<tr>
<td>$k_{rw}^*$</td>
<td>water relative permeability end point</td>
</tr>
<tr>
<td>$L$</td>
<td>core length, inch</td>
</tr>
<tr>
<td>$n_o$</td>
<td>oil relative permeability exponent</td>
</tr>
<tr>
<td>$n_w$</td>
<td>water relative permeability exponent</td>
</tr>
<tr>
<td>$N_B$</td>
<td>Bond number</td>
</tr>
<tr>
<td>$p_{cos}$</td>
<td>surfactant-oil capillary pressure, psi</td>
</tr>
<tr>
<td>$p_{cwo}$</td>
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</tr>
<tr>
<td>$p_o$</td>
<td>oil pressure, psi</td>
</tr>
<tr>
<td>$p_w$</td>
<td>water pressure, psi</td>
</tr>
<tr>
<td>$Q$</td>
<td>produced fluid volume, cc</td>
</tr>
<tr>
<td>$r$</td>
<td>core radius in shape factor equation, ft</td>
</tr>
<tr>
<td>$S$</td>
<td>summation of residuals in optimization</td>
</tr>
<tr>
<td>$SG$</td>
<td>specific gravity</td>
</tr>
<tr>
<td>$S_o$</td>
<td>oil saturation</td>
</tr>
<tr>
<td>$S_w$</td>
<td>water saturation</td>
</tr>
<tr>
<td>$S_{wr}$</td>
<td>residual water saturation in oil flood</td>
</tr>
<tr>
<td>$S_{orw}$</td>
<td>residual oil saturation in waterflood</td>
</tr>
</tbody>
</table>
$S_{ors}$ ................................. residual oil saturation in surfactant flood

T ........................................... Thamama

t ......................................... time, day

$u$ ......................................... Interstitial velocity, $\frac{ft}{day}$

$v$ ......................................... Darcy velocity, $\frac{ft}{day}$

$V$ ......................................... volume, cc

$x$ ......................................... $x$ direction

$x$ ......................................... independent variable in optimization

$y$ ......................................... dependent variable in optimization

$y$ ......................................... $y$ direction in flow equations

$z$ ......................................... vertical axis

**Greek Letters**

$\alpha$ ......................................... capillary coefficient, $psi$

$\beta$ ......................................... vector of unknown parameters

$\gamma$ ......................................... gravity gradient, $\frac{psi}{ft}$

$\lambda$ ......................................... damping factor in optimization

$\lambda$ ......................................... mobility in flow equations, $cp^{-1}$

$\mu$ ......................................... viscosity, $cp$

$\rho$ ......................................... density, $\frac{lb}{ft^3}$

$\sigma$ ......................................... shape factor, $ft^{-2}$

$\sigma_z$ ......................................... shape factor in $z$ direction, $ft^{-2}$
\( \tau \) .................................................. transfer function, \( \text{day}^{-1} \)

\( \phi \) .................................................. porosity

\( \omega \) .................................................. radial speed, \( \frac{\text{radius}}{\text{second}} \)

**Subscripts**

\( c \) .................................................. centrifuge

\( D \) .................................................. dimensionless

\( eff \) .................................................. effective

\( f \) .................................................. fracture

\( gd \) .................................................. gravity drainage

\( i \) .................................................. \( i \)th node in horizontal direction

\( imb \) .................................................. imbibition

\( j \) .................................................. \( j \)th node in vertical direction

\( l \) .................................................. liquid

\( lab \) .................................................. laboratory

\( m \) .................................................. matrix

\( nw \) .................................................. non-wetting

\( o \) .................................................. oil

\( os \) .................................................. oil-surfactant

\( r \) .................................................. residual

\( res \) .................................................. reservoir

\( t \) .................................................. total

\( w \) .................................................. water
φ ......................................................... porous medium

Superscripts

n .............................................................. previous time step

n + 1 ....................................................... current time step

Operators

\n ......................................................... divergence of a vector

\Delta ..................................................... gradient of a scalar function

\partial ..................................................... differentiation
CHAPTER 1
INTRODUCTION

This introductory chapter introduces the reader to the research objectives and methodology.

1.1 Objectives

The objective of this research is to evaluate whether surfactant-augmented waterflooding can improve oil recovery beyond primary production and waterflooding in fractured carbonate reservoirs. Another objective of this dissertation is to provide a better understanding of fluid flow and mass transfer across the fracture-matrix boundaries in fractured carbonate reservoirs, both in low and in high interfacial tension environments.

The contribution of this dissertation includes experimental data obtained using different laboratory procedures and numerical simulation of oil recovery results. As a result of this research, I will demonstrate that surfactant oil recovery is a viable enhanced oil recovery option in fractured carbonate reservoirs when fracture distribution and connectivity are favorable. Oil recovery in fractured carbonate reservoirs is affected by how various forces, e.g., capillary and gravity forces, interact. This is validated when experimental results are analyzed. Specifically, in fractured reservoirs, gravity force plays a key role in driving oil out of the matrix. Unfavorable capillary force and wettability conditions could hinder oil recovery.

The oil recovery assessment was focused on three areas:

1. Matrix-fracture transfer function: The viability of transfer function for oil recovery from fractured carbonates in waterflood and surfactant flood was examined using experimental data and numerical modeling. Two numerical modeling ap-
proaches, used to match oil recovery from fractured carbonate cores, included a zero-dimensional transfer function and a 2-D gridded model.

2. Core-fluid property estimation: The water and oil relative permeability curvatures and end points were determined by history matching.

3. Scaling centrifuge data to the field: The centrifuge oil recovery results were scaled to the field using scaling rules. Field scale oil recovery predictions from these rules were compared with the results from numerical models.

1.2 Methodology

- Laboratory: Experiments are conducted in centrifuge for short cores, and displacement and gravity drainage experiments in long cores. The short cores are 1.5-inch long while the long cores are 12-inch long. The experiments are designed to decipher contributions from rock matrix in presence and absence of fractures, and for imbibition and gravity drainage oil recovery with and without surfactant.

- Fluid flow modeling: Fluid flow in centrifuge experiments is simulated and various flow parameters (i.e., relative permeability and capillary pressure) are calculated by history matching and non-linear regression technique. It will be shown that the model’s theoretical basis is consistent with the physical principles of fluid flow in porous media. Results from these experiments and experience from other fractured carbonate reservoirs, will be the basis for a better understanding of mechanism of enhanced oil production in fractured carbonate reservoirs by micellar solutions.

1.3 Details

Analyze experimental data on fractured carbonate cores, short and long, in low and high interfacial tension environment. Develop a theoretical model to extract
relevant core-fluid properties. Specifically, I will consider the following:

1. Experimental Determine how oil recovery in fractured cores responds to: 1. Brine imbibition under normal gravity force. 2. Forced imbibition under elevated gravity head. 3. Dilute surfactant solutions under normal gravity force and elevated gravity head.


1.4 Workflow

Figure 1.1 shows the workflow chart for this dissertation.

1.5 Work Scope

The following is the details of short and long core experiments and modeling work. 1.5 inch by 1.5 inch short cores o Sources Several carbonate cores from a Silurian dolomite outcrop, a Yates field (West Texas) core, and a Thamama, upper Zakkum field (UAE) core. o Configurations Three types of cores: 1. Some cores are not fractured but are sealed only on the sides. These cores are used as the base for the comparison with following designs. 2. Some cores are fractured and the vertical sides are covered with epoxy glue and Teflon tape while top and bottom faces are not. 3. Some cores are fractured and all surfaces are covered with epoxy glue and Teflon tape except the fracture. o Experiments Selected cores are tested via a complete set of experiments. A set of experiment includes the following sequence:

1. Thin section to review pores and QEM scan to identify mineralogy.

2. Porosity and permeability measurement of cores under various confining stresses.

3. Interfacial tension between oil and surfactant solutions at various temperatures and surfactant concentrations.
• Estimate field scale recovery in fractured carbonate reservoirs
• Estimate enhanced recovery in fractured carbonate reservoirs using surfactants
• Parameter estimation of multiphase flow

Figure 1.1: The experimental and modeling aspect of this dissertation and the expected results.

5. Brine displacement by oil.

6. Spontaneous imbibition by brine under normal gravity.

7. Forced imbibition by brine in centrifuge.

8. Surfactant flood in centrifuge o Number of cores.

9. 12 sets of experiments are conducted on 7 Silurian dolomite cores.

10. 3 sets of experiments on 3 Yates cores.

11. 3 sets of experiments on 3 Thamama cores 1.5 inch by 12 inch long cores.

1.6 Core Sources

Several carbonate cores from a Silurian dolomite outcrop and a Yates field (West Texas) core.

1.7 Core Configurations

Two types of cores:

1. Some cores are not fractured but are sealed only on the sides. These cores are used as the base for the comparison with following designs.

2. Some cores are fractured and the vertical sides are covered with Teflon tape while the top and bottom faces are not.

1.8 Experiments

A complete set of experiment includes following sequence:

1. Porosity and permeability measurement of cores under various confining stresses.

2. Brine saturation of cores.

3. Brine displacement by oil in coreflood set up.
4. Spontaneous imbibition by brine under normal gravity in imbibition cells.
5. Spontaneous imbibition by surfacnat under normal gravity in imbibition cells.
6. Brine flood, and surfactant flood in coreflood set up. Long cores do not necessarily go under the entire the experimental steps explained above. More details will be discussed in the context of this document.

1.8.1 Number of Cores

5 long Silurian dolomite cores and 1 long Yates core will be tested.

1.8.2 Oil, Brine and Surfactant

Silurian dolomite and Yates cores are tested with dead Yates crude and Yates synthetic brine. Thamama cores are tested with dead Thamama crude and brine. S13D (a commercial surfactant by TIORCO and STEPAN) are used for Silurian dolomite and Yates cores. A simple surfactant is used for Thamama cores such as an Ethoxylated Alcohol.

Multiphase flow parameter estimation via experiments o Important characteristics of the fluid flow through porous media such as relative permeability and capillary pressure are obtained through short core experiments. o Due to the limitations on experimental procedures and equipment, we are not able to determine all the flow parameters. For instance, relative permeability of oil during water drainage or water relative permeability during forced imbibition, are quantities which cannot be calculated with the current centrifuge technology. Wherever possible, long core data might be used for estimations.

1.9 Multiphase Flow Parameter Estimation via Experiments

Important characteristics of the fluid flow through porous media such as relative permeability and capillary pressure are obtained through short core experiments. Due to the limitations on experimental procedures and equipment, we are not able
to determine all the flow parameters. For instance, relative permeability of oil during water drainage or water relative permeability during forced imbibition, are quantities which cannot be calculated with the current centrifuge technology. Wherever possible, long core data might be used for estimations.

1.10 Numerical Simulation

A 1-D model for 3-phase flow in centrifuge tests is developed. It models oil recovery from fractured and unfractured cores at different configurations. The model replicates experimental data to examine the consistency of the proposed theories for fractured carbonate cores with physical laws of fluid flow through porous media.

The aforementioned model is extended for low interfacial tension environments including surfactant. The model replicates experimental results. Primary concepts such as surfactant adsorption to the rock matrix and changing the relative permeability end points depending on surfactant concentration are integrated in this model. A simulator is developed to predict core-fluid parameters using Levenberg-Marquardt regression algorithm. This model predicts the core-fluid parameters such as capillary pressure and relative permeability.

1.11 Project Status

Preliminary experiments were conducted with the new centrifuge machine on available samples in the Core Preparation Laboratory in PE Department. Procedure was established to correctly calibrate and run the machine and its accessories to assure accuracy of data.

- 9 complete sets of short core experiments were conducted on Silurian dolomite and another 3 cores are being prepared for the fourth and last test in this series.

- Long core oil recovery experiments were conducted using TIORCO (A NALCO & STEPAN COMPANY) laboratory facilities.
• 3 Yates cores are being prepared for short core experiments.

• 3 Thamama cores are being prepared for short core experiments.

• Numerical code is being developed for two phase flow in the centrifuge for oil-water and oil-surfactant systems in fractured cores.

• Regression algorithm is being coded for two phase flow.

1.12 TimeTable

Figure 1.2 presents the progress to date and proposed plan.

Figure 1.2: Planned Timeline
CHAPTER 2
LITERATURE REVIEW

This chapter reviews literature related to the key objectives of this dissertation in five parts. Enhanced Oil Recovery by surfactant in carbonate reservoirs is discussed in section 2.1. Important concepts such as Bond number are introduced in this part. Concept of transfer function and its evolution from basic formulations to more comprehensive forms, usable in surfactant flooding is discussed and presented in section 2.2. Conventional applications of centrifuge, and the novelty of this research in using centrifuge for fractured cores in surfactant-augmented solutions, are discussed in section 2.3. Scaling laboratory imbibition and gravity drainage oil recovery results to the field, and conventional methods to obtain relative permeability data, are discussed in sections 2.4 and 2.5.

2.1 Chemical Flooding in Fractured Carbonate Reservoirs

Many giant oil and gas reservoirs are naturally fractured carbonates. Carbonate reservoirs hold more than 60% of the world’s oil and 40% of the world’s gas reserves, (Schlumberger, 2008). Carbonate reservoirs own unique challenges for oil recovery forecast and production. Dual porosity and permeability, oil wetness to mixed wetness nature of these reservoirs, and to heterogeneity and low matrix porosity, are the main issues affecting oil production from these reservoirs, (Alvarado & Manrique, 2010; Dennis & Slanden, 1988; Perez et al., 1992).

Enhanced oil recovery (EOR) in carbonates has had more challenges than sandstone reservoirs. Applications of EOR techniques to different lithologies are compared in Figure 2.1. It is seen that carbonate reservoirs have been less subjected to EOR methods in comparison to the sandstone reservoirs, while among three major EOR techniques, gas injection has been favored rather than chemical flooding and thermal
methods. Although chemical treatment of carbonate reservoirs started as early as the 1990s, it was not considered as a major technique for more than a decade. However, the applicability of this method in carbonate reservoirs is getting more attention recently, (Najafabadi et al., 2008; Tabary et al., 2009; Yang & Wadleigh, 2000).

Oil recovery from dilute surfactant injection to the fractured reservoirs has four main challenges including: (1) maximizing surfactant concentration inside fractures, (2) transferring surfactant from fracture to the matrix block, (3) mobilizing oil inside matrix block, and (4) recovering produced oil from fracture network, (Yang & Wadleigh, 2000). Two major mechanisms have been proposed for mobilizing oil inside matrix block are: 1. lowering oil-water interfacial tension (IFT), where buoyancy is the driving force for oil recovery, and 2. altering formation wettability from oil wet to intermediate or water wet, where capillarity is the driving force for oil recovery. Reduction in IFT reduces Bond number, Eq. 2.1, resulting in water imbibition into the rock pores. Invasion of surfactant into the matrix block can lead to wettability alteration; therefore, oil recovery depends on how these forces work together, (Mohanty, 2006). If surfactant alters wettability from oil wet to water wet, reduction in IFT will
slow recovery since capillary force is reduced. If surfactant does not change original oil-wetness, reduction in IFT will promote oil recovery, (Adibhatia & Mohanty, 2007).

Bond number is a dimensionless number representing the ratio of gravity forces to capillary forces, Eq. 2.1, (Du Prey, 1978). It is a measure of the relative contribution of these two forces, where at a “critical” level, will affect residual oil saturation decreases. Figure 2.2 is a plot of the Bond number versus residual wetting saturation obtained by Filoco & Sharma (1998). They presented a capillary desaturation curve for residual oil in oil-wet cores and residual water in water-wet cores.

\[ N_B = \frac{\Delta \rho g L}{\sqrt{\phi k IFT}} \]  \hspace{1cm} (2.1)

where \( \Delta \rho \) is the difference between phase densities, \( g \) acceleration of gravity, \( L \) length, \( \phi \) porosity, \( k \) permeability, and \( IFT \) interfacial tension.

When possible, a critical Bond number should be determined prior conducting experiments. The critical Bond number for wetting phase is \( 10^{-3} \) and for non-wetting phase \( 10^{-5} \) (Lake, 1989). Surfactant decreases IFT (on the order of \( 10^{-3} \)) which moves residual oil saturation to very low values. Residual oil saturation decreases as concentration of surfactant increases, and, consequently, water relative permeability end point increases, (Healy & Reed, 1977; Nelson & Pope, 1978; Stegemeier, 1974; Taber, 1969). Although the exact nature of \( S_{orw} \) reduction is not known, in this work, a nonlinear relationship between \( S_{orw} \) and surfactant concentration was employed. Shifts in \( S_{orw} \) and IFT reduction are the two main parameters used in this thesis to match experimental results with the numerical models of the thesis.

One example of residual saturation versus Bond number is presented in Figure 2.3 by Morrow et al., 1988. For Bond numbers of above 0.01 desaturation of \( S_{orw} \) begins.

Capillary number is a dimensionless number representing the ratio of viscous forces to capillary forces, Eq. 2.2 (Stegemeier, 1977). It is a measure of the relative contribution of these two forces. At a “critical” capillary number the residual oil saturation
Figure 2.2: Effect of core wettability on capillary desaturation for oil-brine systems (Filoco & Sharma, 1998).

Figure 2.3: Residual non-wetting phase vs. inverse of Bond number for air-oil and water-oil system. $N_B^{-1} = \frac{\sigma}{(\rho_w - \rho_o) gr^2}$ where $r$ is the bead size (Morrow et al., 1988).
decreases. Figure 2.2 is a plot of the capillary number versus residual saturation (Stegemeier, 1977). Results from various porous media and fluids have been summarized in this figure.

\[ N_C = \frac{v\mu}{IFT} \]  

(2.2)

where \( v \) is Darcy velocity, \( \mu \) is fluid viscosity and IFT is interfacial tension.

![Figure 2.4: Average experimental recoveries of residual phases (Stegemeier, 1977).](image)

\( S_{orc} \) is the critical residual oil saturation and \( S_{or} \) is residual oil saturation.

### 2.2 Centrifuge

Centrifuge has been widely used since 1950s to evaluate core properties such as residual oil, capillary pressure, wettability characteristics, and relative permeability, (Bentsen & Anli, 1977; Hagoort, 1980; Melrose, 1986; Slobod et al., 1951; Spronsen, 1982; Ward & Morrow, 1987). Much of the interest in the use of centrifuge is determination of relative permeability and capillary pressure (Fleury et al., 2001; Saeedi & Pooladi-Darvish, 2007). Several methods have been proposed to calculate capillary
pressure from centrifuge data (Hassler & Brunner, 1945). Similarly, centrifuge has been used to calculate relative permeability by history matching. Hagoort (1980) gave an analytical method for calculating gravity drainage relative permeability.

Centrifuge is used to simulate large gravity forces as illustrated in Figure 2.5. The magnitude of the simulated gravity force varies as a function of the distance from center of rotation as indicated by Eq. 2.3 through 2.5. Eq. 2.3 is a relationship between the simulated gravity force, speed of rotation, and distance from the center of rotation. The equation indicates that the simulated gravity force increases as distance is increased from the center of rotation. Hence, gravity force is larger toward the bottom of the core in centrifuge experiments (Tiab & Donaldson, 2004).

\[ g_c = \bar{r} \omega^2 \]  \hspace{1cm} (2.3)

\[ \omega = 2\pi \left( \frac{rpm}{60} \right) \]  \hspace{1cm} (2.4)

\[ \bar{r} = \frac{(r + r_1)}{2} \]  \hspace{1cm} (2.5)

Pressure oil-water system is obtained by 2.6 for any distance \( r \) from the center of centrifuge. \( r_1 \) is the distance from center of rotation to the top of the core.

\[ p(r) = p(r_1) + (\rho_w - \rho_o) g_c (r - r_1) \]  \hspace{1cm} (2.6)

For the centrifuge use, 2.6 becomes as 2.7 or 2.8 where \( r_2 \) is the distance from center of centrifuge to the bottom of core and \( r_2 - r_1 = L \).

\[ p(r_2) = p(r_1) + (\rho_w - \rho_o) g_c (r_2 - r_1) \]  \hspace{1cm} (2.7)

or,

\[ p(r_2) - p(r_1) = (\rho_w - \rho_o) g_c L \]  \hspace{1cm} (2.8)

Figure 2.6 illustrates the position of core and fluids in the core holder and cup inside a centrifuge and shows how gravity affects the direction of fluid motion during each cycle. In modern centrifuge designs, the amount of fluid produced is observed by
Figure 2.5: Schematic of a core in the centrifuge.
a digital camera through a window on the bucket, Figure 2.8. A beam of light goes through the receiver cups, so the camera catches the movement of interface between dark colored fluid (oil) and light colored fluid (water or gas). The camera records the position of the interface as a pixel number, which is used to obtain the amount of displaced fluid. Like any other equipment, the centrifuge needs calibration before running experiments. Calibration is conducted on the rotor and camera to update any changes on their mechanical properties. Figure 2.7 Shows the geocentrifuge in Idaho National Lab (INL). The geocentrifuge has an asymmetric beam equipped with a pendulum swinging basket that rotates in a cylindrical steel–concrete enclosure, which offers both centrifuge safety and aerodynamic efficiency during operation. One significant feature of the geocentrifuge is an automatic balancing system. Because many environmental geocentrifuge applications may require fluid movement, this could lead to a change in the center of mass of the sample chamber. Our geocentrifuge will automatically compensate for such shifts during operation. It can rotate at 51–261 rpm and generate gravity of 11-145 times of earth gravity acceleration (INL, April 2011).

In this work, we used the centrifuge and designed appropriate experiments to evaluate the viability of matrix-fracture transfer function in fractured carbonate rocks with and without surfactant in the aqueous phase. To the best knowledge of the author, it is the first time that centrifuge has been used to study fractured cores in surfactant augmented solutions.

2.3 Scaling Rules

Centrifuge has been used for scaling purposes from laboratory to field. Scaling rules for laboratory data to the field for imbibition experiments were studied by Høgnesen et al. (2004); Mattax & Kyte (1962); Standnes (2010) and Zhang et al. (1996), and for gravity drainage experiments by Kyte (1970) and Hagoort (1980). For practical engineering studies, oil recovery scaled times in the field can be calculated from Eq. 2.9, which is derived from a dimensionless time function in a pure gravity
Figure 2.6: Gravity affects fluid both inside the core and surrounding the core. (a) drainage (low density fluid displacing high density fluid), and (b) forced imbibition (high density fluid displacing low density fluid).
drainage model (Hagoort, 1980). Similarly, the scaled reservoir height can be related to the core length by Eq. 2.10. Substituting Eq. 2.10 in Eq. 2.9 yields Eq. 2.11 which can be used to calculate oil recovery times in the field. As an example, for 884 rpm used in a short Silurian core experiments, we obtain \( \frac{g_c}{g} = 145 \). Using this information in Eq. 2.11 for a time of 1 hour in centrifuge, we obtain a scaled reservoir time of 2.4 years. Similarly, using Eq. 2.10, a 1.5-inch long core corresponds to a reservoir height of 18.1 ft.

\[
  t_{res} = \left( \frac{g_c}{g} \right) \frac{L_{res}}{L_{lab}} t_{lab} \\
  L_{res} = \left( \frac{g_c}{g} \right) L_{lab} \\
  t_{res} = \left( \frac{g_c}{g} \right)^2 t_{lab}
\]  

(2.9)  
(2.10)  
(2.11)

The imbibition dimensionless time was initially given by Mattax & Kyte (1962) where it was assumed that core and fluid properties were the same for the core and reservoir rock. This scaling rule was slightly modified by Ma et al. (1995), given by Eq. 2.12. For gravity drainage, Hagoort (1980) presented Eq. 2.13. A more practical
Figure 2.8: Core holder, receiver cups and buckets for (a) drainage (oil displacing water) and (b) imbibition (water displacing oil) tests.
scaling rule is in the form of Eq. 2.15 by Kyte (1970). Eq. 2.10 and Eq. 2.11 can be
derived from 2.13. Viability of the scaling rules can be also established using the
differential equations describing flow in cores and reservoirs.

\[ t_{D,imb} = \left[ \frac{k}{\phi \sqrt{\mu_o \mu_o \mu L^2}} \right] t \]  
(2.12)

\[ t_{D,imb} = \left[ \frac{k k^* (\Delta \rho g)}{\phi^* \mu_o L} \right] t \]  
(2.13)

where,

\[ \phi^* = \phi_m (1 - S_{tr}) \]  
(2.14)

\[ t_{res} = \left( \frac{\mu_{res} k_{lab} IFT_{lab}}{\mu_{lab} k_{res} IFT_{res} \left( \frac{g_c}{g} \right)^2} \right) t_{lab} \]  
(2.15)
Experiments are conducted in a centrifuge and a coreflood apparatus. The centrifuge experiments use cores with dimensions of 1.5 inch in length and 1.5 inch in diameter. The coreflood experiments use cores with dimensions of 12 inch in length and 1.5 inch in diameter. Centrifuge and coreflood tests are designated short core experiments and long core experiments, respectively. A great amount of time is spent in preparation of each core before any test was conducted. Preparation of a core includes cutting it to the exact dimensions, measuring porosity and permeability of the matrix, fracturing the core, measuring the effective permeability of the fractured core, and saturating cores with brine. Cores that are chosen to be used in multiple experiments will go through the cleaning process. Some of the cores will be selected for thin section and QEMSCAN® (Quantitative Evaluation of Minerals by Scanning Electron Microscopy) tests to determine pore characteristics and mineralogy of the matrix.

3.1 Experiments

The laboratory experiments are presented in the following sections:

3.1.1 Centrifuge

A Beckman ultra-fast centrifuge (ACES 200) with a maximum rpm of 16,500 for imbibition and 15,500 rpm for drainage was used for short cores. The centrifuge was capable of running both water drainage (oil displacing water or gas displacing liquid) and imbibition (water displacing oil or gas) experiments. Figure 3.1 shows the centrifuge set up in the PE department. Drainage and imbibition cycles were conducted
with different rotors, core holders, fluid receiver cups and buckets. Figure 2.8 shows
the core holders, fluid receiver cups and buckets used.

![Ultra-fast centrifuge setup](image)

Figure 3.1: The ultra-fast centrifuge set up in the PE department.

### 3.1.2 Coreflood Apparatus

The coreflood apparatus was a Chandler FRT (Formation Response Tester) 6100 model, shown in Figure 3.2. This apparatus consists of a core holder, accumulator, pumps, production collector, and back pressure regulator. It is equipped with several air-controlled and manual valves, pressure transducers, and heating elements around the core holder, which allows for raising the operating temperature to 350°F. The entire system is monitored by a computer, and data is recorded by software in time intervals determined by the user. The plumbing design allows for different flooding scenarios, such as injection or production from top or bottom of the core, top flush and bottom flush. Its maximum confining pressure is 6,000 psi, and maximum pumping pressure is 5,500 psi. The core holder is connected to pressure transducers through pressure taps at different locations along the length of the core to measure
pressure drop, not only across the core, but also at different locations along the core. Spacers with different lengths can be used to allow the use of cores varying in length from 1 inch to 12 inches.

Figure 3.2: Chandler FRT 6100 coreflooding apparatus in TIORCO’s laboratory facilities, Denver, CO.

3.2 Fluids and Cores

The fluids and cores are being used in the experiments are presented in this section.

Table 3.1 and Table 3.2 show properties of the crude and brine used in Silurian dolomite cores (Michigan basin). Brine is synthesized using several salts to represent the brine of a Permian basin, West Texas carbonate field. Oil is the dead crude from the same field.

Table 3.3 and Table 3.4 show properties of the crude and brine used in Thamama cores. Crude and brine are both from the Thamama field. Oil is dead crude from the field.

3.3 Short Cores

The short core preparation and experiments are described in this section.
Table 3.1: West Texas oil properties for the Silurian dolomite core experiments.

<table>
<thead>
<tr>
<th>Oil 0 API</th>
<th>Density (g/cc)</th>
<th>Viscosity (cp)</th>
<th>IFT to deionized water (dynes/cm)</th>
<th>IFT to brine (dynes/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>28.2</td>
<td>0.878</td>
<td>22.5</td>
<td>25</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 3.2: West Texas brine properties for the Silurian dolomite core experiments.

<table>
<thead>
<tr>
<th>Brine</th>
<th>Density (g/cm³)</th>
<th>Viscosity (cp)</th>
<th>NaCl (wt%)</th>
<th>Na₂SO₄ (wt%)</th>
<th>CaCl₂(2H₂O) (wt%)</th>
<th>MgCl₂(6H₂O) (wt%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1.05-1.1</td>
<td>0.48</td>
<td>0.013</td>
<td>0.0996</td>
<td>0.201</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.3: Oil properties for the Thamama core experiments.

<table>
<thead>
<tr>
<th>Oil 0 API</th>
<th>Density (g/cc)</th>
<th>Viscosity (cp)</th>
<th>IFT to brine (dynes/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>0.86</td>
<td>9.4</td>
<td>18.3</td>
</tr>
</tbody>
</table>

Table 3.4: Brine properties for the Thamama core experiments.

<table>
<thead>
<tr>
<th>Brine</th>
<th>Density (g/cc)</th>
<th>Viscosity (cp)</th>
<th>Salinity (wt%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.05</td>
<td>1.12</td>
<td>&gt; 5</td>
<td></td>
</tr>
</tbody>
</table>

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3.3.1 Preparation

Carbonate core plugs studied in this dissertation are from two different sources—a Silurian dolomite outcrop, and Thamama upper Zakum field in UAE. Silurian dolomite cores are tested in both short and long lengths, while cores from Thamama are only available for short lengths. Figure 3.3 presents the flowchart and the sequence of core preparation.

![Flowchart and sequence of core preparation.](image)

Figure 3.3: Flowchart and sequence of core preparation.
3.3.2 Measuring Porosity and Permeability of Core Matrix

Porosity and permeability of all short cores are measured using CMS-300 (core property measurement apparatus), in the PE department. CMS-300 apparatus is able to apply confining stress from 800 psi to 6500 psi. The confining stress is provided by a Nitrogen source gas. Helium gas is injected at 250 psi in the core, and pressure drop is measured for permeability calculations. Grain volume is measured to obtain pore volume by knowing the bulk volume of the core.

3.3.3 Fracturing Cores and Core Designs

Three core configuration will be studied in this research. Figure 3.4 shows three core designs. Figure 3.4(a) is a short unfractured core sealed with epoxy resin on the outer cylindrical surface. Figure 3.4(b) is a fractured core sealed with epoxy resin on the outer cylindrical surface only, and Figure 3.4(c) is a fractured core sealed on the outer cylindrical surface and top and bottom, except for the fracture.

3.3.4 Core Cleaning

Using toluene, cores are cleaned in a Soxhlet setup. Cores are left for several days to weeks to be cleaned in the system. Cores will be used in surfactant flooding will be cleaned by isopropyl alcohol after cleaning by toluene to remove adsorbed surfactant.

3.3.5 Initial Saturation

All cores are initially saturated with brine. Cores are evacuated for 30 minutes and saturated with brine using a vacuum pump. The weight difference before saturation and after saturation with brine is used to calculate pore volume of the cores.

3.3.6 Centrifuge Experiments

A complete cycle of experiment is conducted on the short cores using an imbibition cell and centrifuge. A complete cycle includes water drainage (oil replacing brine) in
Figure 3.4: Three different core configurations used in this work: (a) a whole core sealed with epoxy resin on the outer cylindrical surface only, (b) a fractured core coated with epoxy resin on the outer cylindrical surface only and (c) a fractured core sealed on the outer cylindrical surface, and on top and bottom, while only fracture is open to the flow.
centrifuge, aging cores, spontaneous imbibition of brine in the imbibition cell, forced imbibition (brine replacing oil by gravity) in centrifuge and finally, forced surfactant imbibition (surfactant solution replacing water and oil) in centrifuge. All cores are aged for wettability alteration for four weeks in the same crude at 80°C after oil-displacing-water (drainage) cycle is completed. Cores are left in the beaker during aging process. Recoveries for drainage and forced imbibition cycles are recorded versus time.

3.4 Long Cores

In the remaining parts of this chapter, long core preparation and experiments are described. Long core experiments are conducted in two steps: 1- preparation and 2- static imbibition test. Long core experiments includes only Silurian dolomite carbonate.

3.4.1 Creating Fracture and Aging

Two cores with diameter of 1.5-inch and length of 12-inch were saturated and characterized by injecting synthetic brine into the core in a coreflooding apparatus. One of the cores was cut longitudinally to simulate a reservoir fracture using the same synthetic brine as the lubricant, Figure 3.5. Crude was injected under 400 psi confining pressure to establish initial oil distribution before spontaneous brine imbibition experiment. Brine and oil average saturation were obtained every half an hour. More than 6 PV of oil were injected at 2.4 $\frac{cc}{min}$. After establishing the irreducible water saturation, the core was bathed in the crude oil while aging at 80°C for four weeks to establish formation wettability, Figure 3.6. The same procedure was used on the second long core, except that the core was not fractured. Experiments were conducted on the two cores to compare oil recoveries from the fractured core with the unfractured core. Table 3.5 presents properties of two long cores. Effective permeability is the increased permeability as the result of fracturing.
Table 3.5: Properties of the 12-inch long cores.

<table>
<thead>
<tr>
<th>Sample ID</th>
<th>Configuration</th>
<th>Porosity (%)</th>
<th>Pore volume (cc)</th>
<th>$k_w$ (md)</th>
<th>$k_{eff}$ (md)</th>
<th>Initial oil saturation</th>
</tr>
</thead>
<tbody>
<tr>
<td>LD-7</td>
<td>Fractured</td>
<td>16</td>
<td>53.7</td>
<td>296</td>
<td>737</td>
<td>0.74</td>
</tr>
<tr>
<td>LD11</td>
<td>Unfractured</td>
<td>15</td>
<td>51.3</td>
<td>140</td>
<td>–</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Figure 3.5: (a) unfractured core in the brine and (b) artificially fractured core in the brine.
Figure 3.6: Photographs of cores in process of aging; (a) fractured core after oil injection and (b) the same core immersed in the oil for aging.

3.4.2 Static Imbibition Test for Long Cores

Aging long cores was followed by immersing them in brine for spontaneous imbibition, Figure 3.7. For spontaneous imbibition, both cores were wrapped with several layers of Teflon tape. Then, cores were immersed in the surfactant solution. Top and bottom of cores were exposed to brine and surfactant, but sides were not. A spacer was used beneath the core inside the imbibition cell to allow for gravity effect on the core. Oil recovery was recorded versus time based on the amount of oil produced on top of the imbibition cell and initial oil volume inside the core.
Figure 3.7: Photograph showing produced oil from unfractured core LD11 (left) and fractured core LD7 (right).
CHAPTER 4

NUMERICAL MODELING OF OIL RECOVERY IN FRACTURED AND UNFRACTURED CORES

Two approaches, to simulate laboratory oil recovery from fractured and unfractured cores, are transfer function approach and gridded model approach. In the transfer function approach, cores are treated as a single node while in the gridded model, cores are divided into multiple nodes in the $z$ (vertical) and the $x$ (horizontal) directions. Governing equations for both methods have been described below and in Appendix A.

4.1 Transfer Function Approach

Transfer function concept was introduced in Chapter 2. Material balance of surfactant using transfer function is given by Eq. 4.1, (Al-Kobasi et al., 2009).

$$\tau_w C_s^* = \phi_m \frac{\partial (C_s S_w)}{\partial t} + (1 - \phi_m) S G_{solid} \frac{\partial a}{\partial t} \quad (4.1)$$

The term on the left side of Eq. 4.1 is the surfactant mass transport between matrix and fracture as the result of fluid transfer between fracture and matrix. Terms on the right side of Eq. 4.1 are surfactant accumulation and adsorption of surfactant to the rock. It was assumed that surfactant is carried only by brine and not by oil. By differentiating adsorption term in Eq. 4.2 and substituting it in the finite-difference form of Eq. 4.1, surfactant concentration change inside the core is computed. The details of the model equations and adsorption profile are given in Appendix A.

$$a = \frac{b C_s}{1 + b C_s} a_{max} \quad (4.2)$$

Based on definition of shape factor introduced in Chapter 2, the shape factor for the short and long fractured cores with different boundary conditions are calculated from Eq. 4.3 through Eq. 4.8.
For short cores with open top and bottom:

\[ \sigma = 4 \left[ \frac{1}{L_z^2} + \frac{2}{\pi r^2} \right] = 908 \left( \frac{1}{ft^2} \right) \]  
(4.3)

\[ \sigma_z = 4 \left[ \frac{1}{L_z^2} \right] = 256 \left( \frac{1}{ft^2} \right) \]  
(4.4)

For short cores with only fracture open:

\[ \sigma = 8 \left( \frac{1}{\pi r^2} \right) = 652 \left( \frac{1}{ft^2} \right) \]  
(4.5)

\[ \sigma_z = 4 \left[ \frac{1}{L_z^2} \right] = 256 \left( \frac{1}{ft^2} \right) \]  
(4.6)

And for long cores:

\[ \sigma = 4 \left[ \frac{1}{L_z^2} + \frac{2}{\pi r^2} \right] = 656 \left( \frac{1}{ft^2} \right) \]  
(4.7)

\[ \sigma_z = 4 \left( \frac{1}{L_z^2} \right) = 4 \left( \frac{1}{ft^2} \right) \]  
(4.8)

4.2 Gridded Model Approach

A 2-D gridded model will be developed to simulate oil recovery from fractured cores for the centrifuge experiments. In this model, the core is gridded using \( I_{max} \) number of nodes in vertical direction and \( J_{max} \) number of nodes in horizontal direction. Governing equations for pressure and saturation are solved for waterflood and surfactant flood. Pressure equation yields a system of equations, which are solved implicitly using a linear solver, while saturation equation is solved explicitly for each node. In surfactant flood, surfactant concentration is solved explicitly for each node after pressure and saturation equations are solved. Fluid flow both in the matrix and fracture are controlled by viscous, gravity and capillary forces. The interaction between phases are included via relative permeability and capillary pressure functions.

The boundary conditions for the 2-D modeling are:
**Boundary Condition 1:** vertical side of the core is sealed, and it is considered as a no-flow boundary condition. Top and bottom of core are completely open to the flow. Figure 4.1 shows schematics of a 2-D model with two columns of matrix and a column of fracture between the matrices with top and bottom open to the flow.

**Boundary Condition 2:** vertical side of the core is sealed, and it is considered as a no-flow boundary condition. Top and bottom of core are closed at the matrix interface, but fracture is open to the flow. Figure 4.2 shows schematics of a fractured core with only fracture open to the flow.

The auxiliary equations used in simulations are Eq. 4.9 through Eq. 4.12.

\[ S_w + S_o = 1 \] (4.9)

\[ c_\phi + S_w c_w + S_o c_o = c_t \] (4.10)

\[ \lambda_t = \lambda_w + \lambda_o \] (4.11)

\[ p_o = p_w + p_{cwo} \] (4.12)

The mathematical development, finite difference development of the governing equations and the boundary conditions are given in Appendix A.

### 4.2.1 Pressure Equation

The pressure equations for water phase and oil phase are given by Eq. 4.13 and Eq. 4.14, (Kazemi et al., 1978). Combining these equations, the compact form of working pressure equation becomes Eq. 4.15 and as differential form becomes Eq. 4.15.

\[ \nabla k \lambda_w \left( \nabla p_w - \gamma_w \left( \frac{g_c}{g} \right) \nabla D \right) = \phi S_w (c_\phi + c_w) \frac{\partial p_w}{\partial t} + \phi \frac{\partial S_w}{\partial t} \] (4.13)
Figure 4.1: Gridding structure for modeling oil recovery from fractured cores in centrifuge where top and bottom are completely open.
Figure 4.2: Gridding structure for modeling oil recovery from fractured cores in centrifuge where only fracture is open to the flow.
\[ \nabla . k_\lambda \left( \nabla p_o - \gamma_o \left( \frac{g_c}{g} \right) \nabla D \right) = \phi S_o (c_\phi + c_o) \frac{\partial p_o}{\partial t} + \phi \frac{\partial S_o}{\partial t} \quad (4.14) \]

\[ \nabla . k \left( \lambda_t \nabla p_w + \lambda_o \nabla p_{cwo} - (\lambda_w \gamma_w + \lambda_o \gamma_o) \left( \frac{g_c}{g} \right) \nabla D \right) = \phi c_i \frac{\partial p_w}{\partial t} + \phi (S_o c_\phi + S_o c_o) \frac{\partial p_{cwo}}{\partial t} \quad (4.15) \]

\[ \frac{\partial}{\partial z} \left\{ k \left( \frac{\partial p_w}{\partial z} + \lambda_o \frac{\partial p_{cwo}}{\partial z} - (\lambda_w \gamma_w + \lambda_o \gamma_o) \left( \frac{g_c}{g} \right) \frac{\partial D}{\partial z} \right) \right\} + \frac{\partial}{\partial x} \left\{ k \left( \lambda_t \frac{\partial p_w}{\partial x} + \lambda_o \frac{\partial p_{cwo}}{\partial x} \right) \right\} = \phi c_i \frac{\partial p_w}{\partial t} + \phi (S_o c_\phi + S_o c_o) \frac{\partial p_{cwo}}{\partial t} \quad (4.16) \]

### 4.2.2 Saturation Equation

The saturation equation is defined by Eq. 4.17 and Eq. 4.18, (Kazemi et al., 1978):

\[ \nabla . k_\lambda \left( \nabla p_w - \gamma_w \left( \frac{g_c}{g} \right) \nabla D \right) = \phi S_w (c_\phi + c_w) \frac{\partial p_w}{\partial t} + \phi \frac{\partial S_w}{\partial t} \quad (4.17) \]

\[ \frac{\partial}{\partial z} \left( k \lambda_w \frac{\partial p_w}{\partial z} - k \lambda_w \gamma_w \left( \frac{g_c}{g} \right) \frac{\partial D}{\partial z} \right) + \frac{\partial}{\partial x} \left( k \lambda_w \frac{\partial p_w}{\partial x} \right) = \phi S_w (c_\phi + c_w) \frac{\partial p_w}{\partial t} + \phi \frac{\partial S_w}{\partial t} \quad (4.18) \]
4.2.3 **Surfactant Equation**

The surfactant concentration equation is given by Eq. 4.19. The finite-difference form of this equation is given in Appendix A.

\[
\nabla k \lambda_w C_s \left( \nabla p_w - \gamma_w \left( \frac{g_c}{g} \right) \nabla D \right) = \phi C_s S_w (c_\phi + c_w) \frac{\partial p_w}{\partial t} \\
+ \phi \frac{\partial (C_s S_w)}{\partial t} + (1 - \phi_m) S G_{solid} \frac{\partial a}{\partial t}
\]

(4.19)

4.3 **Core-Fluid Properties Using Regression Analysis**

A 1-D, 2-phase flow model was developed for a whole core (unfractured). This model was used along with the well known Levenberg-Marquardt regression method, to calculate relative permeability parameters. The conventional core properties such as porosity and absolute permeability, in addition to oil recovery data from waterflood experiments were input data to the regression algorithm.

4.3.1 **1-D Fluid Flow Model**

The mathematical model was built for an unfractured core based on the governing equations, in Eq. 4.15 and Eq. 4.17. Schematics of the model is shown in Figure 4.3. In this model, core is gridded only in z direction where gravity affects production. Based on the potential profile, water is entered from the bottom of the core (farther from the center of rotation), and oil and water exit from the top of the core. The same logic is applicable during surfactant flood. Inside the centrifuge, water surrounding the core has higher potential than the oil and water inside the core. If oil pressure inside the core is assumed as a constant line during the replacement of fluids, in an oil-wet system, water pressure is increased. Hence, the water pressure line moves until it becomes a line parallel to the original water pressure inside the core.
Figure 4.3: Initial and boundary conditions for the 1-D model of waterflood using a centrifuge.
4.3.2 Nonlinear Regression Algorithm

Nonlinear regression finds a function \( f \) that fits the sets of data points \((x_i, y_i)\) in the least square sense. \( x_i \) is an independent variable, and \( y_i \) is a dependent or observed variable in Cartesian coordinates. The unknown parameters in \( f \) are usually expressed as a vector like \( \vec{\beta} \). Nonlinearity is referred to unknown parameters and not to the independent variables, \( x_i \). The Levenberg-Marquardt Algorithm (LMA) has been proven as an effective and popular method. LMA is an iterative method and needs an initial estimate for the regression parameters. LMA is a relatively robust method in comparison with Gauss-Newton approach, while it may be somewhat slower (Douglas & Watts, 1988; Press et al., 2001). If the number of unknown parameters is \( N \) and the number of data points (measurements) is \( M \), then the residual for each data point is defined as Eq. 4.20, and the summation of square residuals is defined as Eq. 4.21. \( N \) must be less than \( M \) to have adequate equations for unknowns to be solved.

\[
\begin{align*}
  r_i &= y_i - f(x_i, \beta) ; i = 1, 2, 3..., M \\
  S &= \sum_{i=1}^{M} r_i^2 = \sum_{i=1}^{M} (y_i - f(x_i, \beta))^2
\end{align*}
\tag{4.20}
\tag{4.21}
\]

LMA minimizes \( S \). An initial estimate is required for the vector of unknown parameters, \( \beta \). If the initial estimate is too far from the true value, convergence can be an issue. First order Taylor series expansion for \( \beta \) yields Eq. 4.22.

\[
  f(x_i, \beta + \Delta \beta) = f(x_i, \beta) + \sum_{i=1}^{M} \frac{\partial f(x_i, \beta)}{\beta_j} \Delta \beta_j = f(x_i, \beta) + J_i \Delta \beta
\tag{4.22}
\]

By defining Jacobian matrix as Eq. 4.23, Eq. 4.22 can also be expressed as Eq. 4.24. \( J_i \) is the entire row \( i \) in Jacobian matrix.
\[ J_{M,N} = \begin{bmatrix} \frac{\partial f(x_1, \beta)}{\partial \beta_1} & \cdots & \frac{\partial f(x_1, \beta)}{\partial \beta_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial f(x_M, \beta)}{\partial \beta_1} & \cdots & \frac{\partial f(x_M, \beta)}{\partial \beta_N} \end{bmatrix} \]  
(4.23)

\[ f(x_i, \beta + \Delta \beta) - f(x_i, \beta) = J_i^i \Delta \beta \]  
(4.24)

\[ S \text{ in Eq. 4.21 is minimum if its gradients with respect to } \beta \text{ are zero. Algebraic implementations leads to Eq. 4.25.} \]

\[ (J^T J) \Delta \beta = J^T (y - f(x, \beta)) \]  
(4.25)

\( J^T \) is transpose of \( J \). The unknown parameters vector, \( \beta \), is updated by a new estimation in every iteration. Levenberg added a new parameter, \( \lambda \), called “damping factor”, which is adjusted in every iteration, Eq. 4.26. \( I \) is the identity matrix. If reduction of \( S \) is rapid, a smaller value can be used, whereas if an iteration gives insufficient reduction in the residual, \( \lambda \) can be increased. Levenberg’s algorithm has the disadvantage that if the value of damping factor is large, then \( \Delta \beta \) will remain unchanged. Marquardt provided a method that can scale each component of gradient according to the curvature so that there is larger movement along the directions where the gradient is smaller. This avoids slow convergence in the direction of small gradient. Marquardt replaced the identity matrix with diagonal of Hessian matrix, \( J^T J \), resulting in the final version of Levenberg-Marquardt algorithm defined as Eq. 4.27, (Douglas & Watts, 1988).

\[ (J^T J + \lambda I) \Delta \beta = J^T (y - f(x, \beta)) \]  
(4.26)

\[ (J^T J + \lambda \text{diag}(J^T J))(\beta^{k+1} - \beta^k) = J^T (y - f(x, \beta)) \]  
(4.27)

LMA can be summarized as the following steps:

1. Give an initial estimate of the parameters.

2. Pick a modest value for \( \lambda \) like \( \lambda = 0.001 \).
3. Compute \((y - f(x, \beta))\) using experimental data.

4. Solve Eq. 4.27 for \(\Delta \beta\) and calculate for \(\beta^{k+1}\).

5. Compute \(S_{\text{new}}\) with \(\beta^{k+1}\).

6. If \(S_{\text{new}} > S_{\text{old}}\), increase \(\lambda\) by factor of 10 and go back to step 4 without updating \(\beta^k\).

7. If \(S_{\text{new}} \leq S_{\text{old}}\), decrease \(\lambda\) by a factor of 10 and update \(\beta^k\) with \(\beta^{k+1}\), go back to step 3.

8. Check for convergence.

In this work, the vector of unknown parameters is water and oil relative permeability curvatures \((n_w, n_o)\) and water and oil relative permeability end points \((k_{rw}^*, k_{row}^*)\). The full vector of unknowns in this work is in the form of

\[
\begin{bmatrix}
  n_w \\
  n_o \\
  k_{rw}^* \\
  k_{row}^*
\end{bmatrix}
\]

Dependent and independent variables are oil recovery and time data points respectively, and functionality, \(f\) is expressed through numerical modeling of oil recovery using pressure-saturation equations. Figure 4.4 is the flowchart of the LMA code used in this research.
Figure 4.4: Core-fluid parameters were obtained using a 1-D simulator and LMA regression algorithm. Simulator solves pressure and saturation equations. In the regression part, relative permeability parameters are calculated and updated.
REFERENCES CITED


APPENDIX - FINITE-DIFFERENCE MODELING

A.1 Pressure Equation

Pressure equation for water and oil in a two-phase system in the centrifuge are defined in Eq. A.1 and Eq. A.2, (Kazemi et al., 1978). Summation of these two equations gives the general pressure equation expressed in water pressure, Eq. A.3.

\[
\nabla \cdot k w \left( \nabla p_w - \gamma_w \left( \frac{g_c}{g} \right) \nabla D \right) = \phi S_w (c_\phi + c_w) \frac{\partial p_w}{\partial t} + \phi \frac{\partial S_w}{\partial t} \tag{A.1}
\]

\[
\nabla \cdot k o \left( \nabla p_o - \gamma_o \left( \frac{g_c}{g} \right) \nabla D \right) = \phi S_o (c_\phi + c_o) \frac{\partial p_o}{\partial t} + \phi \frac{\partial S_o}{\partial t} \tag{A.2}
\]

\[
\nabla \cdot \left( \lambda_t \nabla p_w + \lambda_o \nabla P_{cwo} - (\lambda_w \gamma_w + \lambda_o \gamma_o) \left( \frac{g_c}{g} \right) \nabla D \right) = \phi c_t \frac{\partial p_w}{\partial t} + \phi (S_o c_\phi + S_o c_o) \frac{\partial p_{cwo}}{\partial t} \tag{A.3}
\]

Differential form of pressure equation in two dimensions of z and x is Eq. A.4.

\[
\frac{\partial}{\partial z} \left\{ \kappa \left( \lambda_t \frac{\partial p_w}{\partial z} + \lambda_o \frac{\partial p_{cwo}}{\partial z} - (\lambda_w \gamma_w + \lambda_o \gamma_o) \left( \frac{g_c}{g} \right) \frac{\partial D}{\partial z} \right) \right\} + \frac{\partial}{\partial x} \left\{ k \left( \lambda_t \frac{\partial p_w}{\partial x} + \lambda_o \frac{\partial p_{cwo}}{\partial x} \right) \right\} = \phi c_t \frac{\partial p_w}{\partial t} + \phi (S_o c_\phi + S_o c_o) \frac{\partial p_{cwo}}{\partial t} \tag{A.4}
\]

Eq. A.4 in finite difference form is Eq. A.5.
\[
VR_{i,j} \left( \left( \frac{k\lambda_t}{\Delta z} \right)_{i,j+\frac{1}{2}}^{n} (p_{iw,i,j+1}^{n+1} - p_{iw,i,j}^{n+1}) - \left( \frac{k\lambda_t}{\Delta z} \right)_{i,j-\frac{1}{2}}^{n} (p_{iw,i,j}^{n+1} - p_{iw,i,j-1}^{n+1}) \right) + \left( \frac{k\lambda_o}{\Delta z} \right)_{i,j+\frac{1}{2}}^{n} (p_{cwo,i,j+1}^{n} - p_{cwo,i,j}^{n}) - \left( \frac{k\lambda_o}{\Delta z} \right)_{i,j-\frac{1}{2}}^{n} (p_{cwo,i,j}^{n} - p_{cwo,i,j-1}^{n}) \right)
- \left( \frac{k}{\Delta z} (\lambda_w^2 + \lambda_o^2) \right) \left( \frac{g_c}{g} \right)_{i,j+\frac{1}{2}}^{n} (D_{i,j+1} - D_{i,j}) + \left( \frac{k}{\Delta z} (\lambda_w^2 + \lambda_o^2) \right) \left( \frac{g_c}{g} \right)_{i,j-\frac{1}{2}}^{n} (D_{i,j} - D_{i,j-1})
+ \left( \frac{k}{\Delta z} (\lambda_w^2 + \lambda_o^2) \right) \left( \frac{g_c}{g} \right)_{i,j+\frac{1}{2}}^{n} (p_{cwo,i,j+1}^{n+1} - p_{cwo,i,j}^{n+1}) - \left( \frac{k}{\Delta z} (\lambda_w^2 + \lambda_o^2) \right) \left( \frac{g_c}{g} \right)_{i,j-\frac{1}{2}}^{n} (p_{cwo,i,j}^{n+1} - p_{cwo,i,j-1}^{n+1})
\]

\[= VR_{i,j} \left( \phi_{ct} \frac{p_{w,i,j}^{n+1} - p_{w,i,j}^{n}}{\Delta t} \right)\]

### A.2 Transmissibilities

Total transmissibilities in z direction are defined as Eq. A.6 and Eq. A.7.

\[T_{t,z,i,j+\frac{1}{2}}^{n} = VR_{i,j} \left( \frac{k\lambda_t}{\Delta z} \right)_{i,j+\frac{1}{2}}^{n} \] (A.6)
\[T_{t,z,i,j-\frac{1}{2}}^{n} = VR_{i,j} \left( \frac{k\lambda_t}{\Delta z} \right)_{i,j-\frac{1}{2}}^{n} \] (A.7)

Total transmissibilities in x direction are defined as Eq. A.8 and Eq. A.9.

\[T_{t,x,i+\frac{1}{2},j}^{n} = VR_{i,j} \left( \frac{k\lambda_t}{\Delta x} \right)_{i+\frac{1}{2},j}^{n} \] (A.8)
\[T_{t,x,i-\frac{1}{2},j}^{n} = VR_{i,j} \left( \frac{k\lambda_t}{\Delta x} \right)_{i-\frac{1}{2},j}^{n} \] (A.9)

Oil transmissibilities in z direction are defined as Eq. A.10 and Eq. A.11.

\[T_{o,z,i,j+\frac{1}{2}}^{n} = VR_{i,j} \left( \frac{k\lambda_o}{\Delta z} \right)_{i,j+\frac{1}{2}}^{n} \] (A.10)
\[ T_{o,z,i,j}^{n} = \frac{VR_{i,j}}{\Delta z_{i,j}} \left( \frac{k\lambda_o}{\Delta z} \right)^{n}_{i,j-\frac{1}{2}} \quad (A.11) \]

Oil transmissibilities in x direction are defined as Eq. A.12 and Eq. A.13;

\[ T_{o,x,i+\frac{1}{2},j}^{n} = \frac{VR_{i,j}}{\Delta x_{i,j}} \left( \frac{k\lambda_o}{\Delta x} \right)^{n}_{i+\frac{1}{2},j} \quad (A.12) \]

\[ T_{o,x,i-\frac{1}{2},j}^{n} = \frac{VR_{i,j}}{\Delta x_{i,j}} \left( \frac{k\lambda_o}{\Delta x} \right)^{n}_{i-\frac{1}{2},j} \quad (A.13) \]

Substituting transmissibilities in Eq. A.5 and collecting terms for pressures gives the final form of pressure equation, Eq. A.14.

\[ T_{n,t,z,i,j}^{n} + 1 p_{n+1,w,i,j}^{n} = \left( T_{t,z,i,j+\frac{1}{2}}^{n} + T_{t,z,i,j-\frac{1}{2}}^{n} + T_{t,x,i+\frac{1}{2},j}^{n} + T_{t,x,i-\frac{1}{2},j}^{n} + \frac{VR_{i,j} \phi c_t}{\Delta t} \right) p_{w,i,j}^{n+1} + \]

\[ T_{n,t,x,i}^{n} \frac{p_{cwo,i,j}^{n}}{\Delta t} - T_{n,t,x,i}^{n} \frac{p_{cwo,i,j}^{n-1}}{\Delta t} + \]

\[ T_{n,x,i+\frac{1}{2},j}^{n} \frac{p_{cwo,i+1,j}^{n}}{\Delta t} - T_{n,x,i-\frac{1}{2},j}^{n} \frac{p_{cwo,i-1,j}^{n}}{\Delta t} + \]

\[ \frac{VR_{i,j} \phi c_t}{\Delta t} \]

By defining the following terms, Eq. A.15 through Eq. A.20, the final form of matrix to be solved is Eq. A.21.

\[ E_{i,j} = - \left( T_{t,z,i,j+\frac{1}{2}}^{n} + T_{t,z,i,j-\frac{1}{2}}^{n} + T_{t,x,i+\frac{1}{2},j}^{n} + T_{t,x,i-\frac{1}{2},j}^{n} + \frac{VR_{i,j} \phi c_t}{\Delta t} \right) \quad (A.15) \]

\[ A_{i,j} = T_{t,x,i-\frac{1}{2},j}^{n} \quad (A.16) \]

\[ B_{i,j} = T_{t,x,i+\frac{1}{2},j}^{n} \quad (A.17) \]
\[ C_{i,j} = T_{i,z,i,j + \frac{1}{2}}^n \]  
(A.18)

\[ F_{i,j} = T_{i,z,i,j - \frac{1}{2}}^n \]  
(A.19)

\[ R_{i,j} = \]  
(A.20)

\[ - T_{o,z,i,j + \frac{1}{2}}^n (p_{cwo,i,j+1}^n - p_{cwo,i,j}^n) - T_{o,z,i,j - \frac{1}{2}}^n (p_{cwo,i,j-1}^n - p_{cwo,i,j}^n) \]

\[ - \left( \frac{k(\lambda_w \gamma_w + \lambda_o \gamma_o)}{\Delta z} \right) \left( \frac{g_c}{g} \right)_{i,j+\frac{1}{2}}^n (D_{i,j+1} - D_{i,j}) \]

\[ + \left( \frac{k(\lambda_w \gamma_w + \lambda_o \gamma_o)}{\Delta z} \right) \left( \frac{g_c}{g} \right)_{i,j-\frac{1}{2}}^n (D_{i,j} - D_{i,j-1}) \]

\[ - T_{o,x,i + \frac{1}{2},j}^n (p_{cwo,i+1,j}^n - p_{cwo,i,j}^n) - T_{o,x,i - \frac{1}{2},j}^n (p_{cwo,i-1,j}^n - p_{cwo,i,j}^n) \]

\[ - VR_{i,j} \phi_c p_{w,i,j}^n \quad \Delta t \]

\[
\begin{bmatrix}
E_{1,1} & B_{1,1} & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
\vdots & \ddots & \cdots & \vdots & \cdots & 0 & 0 & 0 & 0 \\
0 & \cdots & \ddots & 0 & 0 & \cdots & 0 & 0 & 0 \\
\vdots & \cdots & 0 & \ddots & \cdots & \vdots & \cdots & 0 & 0 \\
0 & F_{i,j} & 0 & A_{i,j} & E_{i,j} & B_{i,j} & 0 & C_{i,j} & 0 \\
0 & 0 & \cdots & 0 & \cdots & \ddots & 0 & 0 & \cdots \\
0 & 0 & 0 & \cdots & 0 & \cdots & \ddots & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0 & \cdots & \ddots & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\
\end{bmatrix}^n
\]

\[
\begin{bmatrix}
p_{1,1} \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
p_{i,j} \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{bmatrix}^{n+1}
\times
\begin{bmatrix}
R_{1,1} \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
R_{i,j} \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{bmatrix}^n
\]

\[
\begin{bmatrix}
p_{t_{\text{max},j_{\text{max}}}^n} \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{bmatrix}
\times
\begin{bmatrix}
R_{t_{\text{max},j_{\text{max}}}^{n+1}} \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{bmatrix}
\]

(A.21)
A.3 Saturation Equation

Saturation equation for water in a two-phase system is defined as Eq. A.22, (Kazemi et al., 1978). Expansion of Eq. A.22 in two dimensions of z and x yields Eq. A.23. Finite difference of Eq. A.23 and collecting terms for $S_{w,i,j}^{n+1}$ results in Eq. A.24.

\[
\nabla \cdot \lambda_w \left( \nabla p_w - \gamma_w \left( \frac{g_c}{g} \right) \nabla D \right) = \phi S_w \left( c_\phi + c_w \right) \frac{\partial p_w}{\partial t} + \phi \frac{\partial S_w}{\partial t} \quad (A.22)
\]

\[
\frac{\partial}{\partial z} \left( k \lambda_w \frac{\partial p_w}{\partial z} - k \lambda_w \gamma_w \left( \frac{g_c}{g} \right) \frac{\partial D}{\partial z} \right) + \frac{\partial}{\partial x} \left( k \lambda_w \frac{\partial p_w}{\partial x} - k \lambda_w \gamma_w \left( \frac{g_c}{g} \right) \frac{\partial D}{\partial x} \right) = \phi S_w \left( c_\phi + c_w \right) \frac{\partial p_w}{\partial t} + \phi \frac{\partial S_w}{\partial t} \quad (A.23)
\]
\[
\left( \frac{VR_{i,j} \phi}{\Delta t} \right) S_{w,i,j}^{n+1} = VR_{i,j} \left\{ \left( \frac{k \lambda_w}{\Delta z} \right)^n_{i,j+\frac{1}{2}} \left( p_{w,i,j+1}^{[n+1]} - p_{w,i,j}^{[n+1]} \right) \right. \\
- \left. \left( \frac{k \lambda_w}{\Delta z} \right)^n_{i,j-\frac{1}{2}} \left( p_{w,i,j}^{[n+1]} - p_{w,i,j-1}^{[n+1]} \right) \right\} \\
- \left( \frac{k \lambda_w}{\Delta z} \right)^n_{i,j+\frac{1}{2}} \left( g_c \right)^n g \left( D_{i,j+1} - D_{i,j} \right) \\
- \left( \frac{k \lambda_w}{\Delta z} \right)^n_{i,j-\frac{1}{2}} \left( g_c \right)^n g \left( D_{i,j-1} - D_{i,j} \right) \\
+ VR_{i,j} \left\{ \left( \frac{k \lambda_w}{\Delta x} \right)^n_{i+\frac{1}{2},j} \left( p_{w,i+1,j}^{[n+1]} - p_{w,i,j}^{[n+1]} \right) \right. \\
- \left. \left( \frac{k \lambda_w}{\Delta x} \right)^n_{i,j-\frac{1}{2}} \left( p_{w,i,j}^{[n+1]} - p_{w,i-1,j}^{[n+1]} \right) \right\} \\
- VR_{i,j} \phi S_w \left( c_\phi + c_w \right) \frac{p_{w,i,j}^{[n+1]} - p_{w,i,j}^n}{\Delta t} + VR_{i,j} \phi S_w \left( c_\phi + c_w \right) \frac{p_{w,i,j}^{[n+1]} - p_{w,i,j}^n}{\Delta t} = S_{w,i,j}^{n+1}
\]

### A.4 Surfactant Equation

Surfactant equation is defined as Eq. A.25. The last term in the right hand side of this equation is the adsorption term. Adsorption equation is defined in the form of Langmuir equation, Eq. A.26. Using chain rule, Eq. A.27, Eq. A.28 and expansion of Eq. A.25 in two dimensions of z and x yields in Eq. A.29. Finite difference form of Eq. A.29 and collecting terms for \( C_{s,i,j}^{n+1} \) results in Eq. A.30 and Eq. A.31.

\[
\nabla . k C_s \lambda_w \left( \nabla p_w - \gamma_w \left( \frac{g_c}{g} \right) \nabla D \right) = \phi C_s S_w \left( c_\phi + c_w \right) \frac{\partial p_w}{\partial t} + \phi \frac{\partial (C_s S_w)}{\partial t} \left( 1 - \varphi_m \right) S G_{solid} \frac{\partial a}{\partial t} \tag{A.25}
\]

\[
\phi = \frac{b C_s}{1 + b C_s a_{max}} \tag{A.26}
\]
\[
\frac{\partial a}{\partial t} = \frac{\partial a}{\partial C_s} \frac{\partial C_s}{\partial t} \quad \text{(A.27)}
\]

\[
\frac{\partial a}{\partial C_s} = \frac{b}{(1 + bC_s)^2 a_{max}} \quad \text{(A.28)}
\]

\[
\frac{\partial}{\partial z} \left( kC_s \lambda_w \frac{\partial p_w}{\partial z} - kC_s \lambda_w \gamma_w \left( \frac{g_c}{g} \right) \frac{\partial D}{\partial z} \right) + \frac{\partial}{\partial x} \left( kC_s \lambda_w \frac{\partial p_w}{\partial x} - k\lambda_w \gamma_w \left( \frac{g_c}{g} \right) \frac{\partial D}{\partial x} \right) = \phi C_s S_w (c_\varphi + c_w) \frac{\partial p_w}{\partial t} + \phi \frac{\partial (C_s S_w)}{\partial t} + \frac{b}{(1 + bC_s)^2 a_{max}} \frac{\partial C_s}{\partial t} \quad \text{(A.29)}
\]

\[
\frac{VR_{i,j}}{\Delta z_{i,j}} \left\{ \left( \frac{kC_s \lambda_w}{\Delta z} \right)^n_{i,j+\frac{1}{2}} \left( p^{[n+1]}_{w,i,j+1} - p^{[n+1]}_{w,i,j} \right) - \left( \frac{kC_s \lambda_w}{\Delta z} \right)^n_{i,j-\frac{1}{2}} \left( p^{[n+1]}_{w,i,j} - p^{[n+1]}_{w,i,j-1} \right) - \left( \frac{kC_s \lambda_w \gamma_w}{\Delta z} \right)^n_{i,j+\frac{1}{2}} \left( \frac{g_c}{g} \right) \left( D_{i,j+1} - D_{i,j} \right) \right. \\
+ \left. \frac{VR_{i,j}}{\Delta x_{i,j}} \left\{ \left( \frac{k\lambda w}{\Delta x} \right)^n_{i+\frac{1}{2},j} \left( p^{[n+1]}_{w,i+1,j} - p^{[n+1]}_{w,i,j} \right) - \left( \frac{k\lambda w}{\Delta x} \right)^n_{i-\frac{1}{2},j} \left( p^{[n+1]}_{w,i,j} - p^{[n+1]}_{w,i,j-1} \right) \right\} \right\} = \frac{\phi \left( c_\varphi + c_w \right) S_w}{\Delta t} \left( \frac{p^{[n+1]}_{w,i,j} - p^{[n]}_{w,i,j}}{\Delta t} \right) + \frac{VR_{i,j} \phi S_{w,i,j}^{[n+1]} - p^{[n]}_{w,i,j}}{\Delta t} \\
+ \frac{VR_{i,j} \phi S_{w,i,j}^{[n+1]} - p^{[n]}_{w,i,j}}{\Delta t} - \frac{VR_{i,j} \phi S_{w,i,j}^{[n]} - p^{[n]}_{w,i,j}}{\Delta t} + \frac{(1 - \phi_m) S_{solid} \frac{b}{(1 + bC_s)^2 a_{max}}}{\Delta t} \left( C_{s,i,j}^{[n+1]} - C_{s,i,j}^{[n]} \right) \quad \text{(A.30)}
\]
\[
\left( VR_{i,j} \phi S_{w,i,j}^{[n+1]} + (1 - \varphi_m) SG_{solid} \frac{b}{(1 + bC_s)^2 a_{max}} \right) \frac{C_{s,i,j}^{n+1}}{\Delta t} = (A.31)
\]

\[
\begin{align*}
VR_{i,j} & \left\{ \left( \frac{kC_s \lambda_w}{\Delta z} \right)_{i,j+\frac{1}{2}}^n \left( p_{w,i,j+1}^{[n+1]} - p_{w,i,j}^{[n+1]} \right) \\
& - \left( \frac{kC_s \lambda_w}{\Delta z} \right)_{i,j-\frac{1}{2}}^n \left( p_{w,i,j}^{[n+1]} - p_{w,i,j-1}^{[n+1]} \right) \\
& - \left( \frac{kC_s \lambda_w \gamma_w}{\Delta z} \left( \frac{g_c}{g} \right) \right)_{i,j+\frac{1}{2}}^n (D_{i,j+1} - D_{i,j}) \\
& - \left( \frac{kC_s \lambda_w \gamma_w}{\Delta z} \left( \frac{g_c}{g} \right) \right)_{i,j-\frac{1}{2}}^n (D_{i,j-1} - D_{i,j}) \right\} \\
+ VR_{i,j} & \left\{ \left( \frac{k \lambda_w}{\Delta x} \right)_{i+\frac{1}{2},j}^n \left( p_{w,i+1,j}^{[n+1]} - p_{w,i,j}^{[n+1]} \right) \\
& - \left( \frac{k \lambda_w}{\Delta x} \right)_{i-\frac{1}{2},j}^n \left( p_{w,i,j}^{[n+1]} - p_{w,i-1,j}^{[n+1]} \right) \right\} \\
- VR_{i,j} & \phi S_{w} (c_\phi + c_w) \frac{p_{w,i,j}^{[n+1]} - p_{w,i,j}^{[n]}}{\Delta t} \\
& + VR_{i,j} \phi S_{w,i,j}^{[n]} \frac{C_{s,i,j}^{n}}{\Delta t} \\
+ (1 - \varphi_m) & SG_{solid} \frac{b}{(1 + bC_s)^2 a_{max}} \frac{C_{s,i,j}^{n}}{\Delta t}
\end{align*}
\]